LECTURE NOTES: 4-9 ANTIDERIVATIVES

MOTIVATING IDEAS:

1. If $s(t)$ give the position of an object at each time t, then $s'(t)$ is ________________________ and

   $s''(t)$ is ________________________.

2. If $P(t)$ is the number of individuals in a population at each time t, then $P'(t)$ is ________________________.

Definition: Given a function $f(x)$, any function $F(x)$ such that

is called ________________________.

EXAMPLE: Find three different antiderivatives of $f(x) = x^2$.

QUESTION 1: How to any two antiderivatives of $f(x) = x^2$ differ?

QUESTION 2: How can you characterize *all* antiderivatives of $f(x) = x^2$ simultanously? Explain what your expression means.

QUESTION 3: Fill in the blank:

Theorem: If F is an anti-derivative of f on an interval I, then the most general anti-derivative of f on I is

QUESTION 4: Do you really believe this Theorem? Here's a check. It's easy to see that $F(x) = x^2 - \pi$ is an antiderivative of $f(x) = 2x$. (Right?) Apply the Theorem in Question 3 to this choice of F.

QUESTION 5: Describe the family of antiderivatives of $f(x) = x^2$ *geometrically*.

PRACTICE PROBLEMS:

1. Minnie Mouse says that $F(x) = 5x^{2/3} + x + \sqrt{2}$ is an antiderivative of $f(x) = \frac{10+3\sqrt[3]{x}}{3\sqrt[3]{x}}$. Find the *most efficient way* to determine if she is correct.

2. Fill in the table below. Assume n and a are fixed constants.

Function	Particular Anti-derivative	Function	Particular Anti-derivative
x^n $(n \neq -1)$		$\frac{1}{\sqrt{1-x^2}}$	
$\cos x$		$\frac{1}{1+x^2}$	
$\sin x$		$\frac{1}{x}$	
$\sec^2 x$		e^x	
$\csc^2 x$		a^x	
$\sec x \tan x$		$\csc x \cot x$	

3. Assuming $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$, fill in the blanks below.

   (a) For any constant a, the general antiderivative of $af(x)$ is ____________________.

   (b) The general antiderivative of $f(x) + g(x)$ is ____________________.

4. Find the most general antiderivative of each function below and then check your answer.

(a) $f(x) = x^{20} + 4x^{10} + 8$

(b) $f(t) = \frac{5 \sec t \tan t}{3} - 4 \sin t - \frac{1}{t^2} + e^2$

(c) $g(x) = x(2x^5 + \sqrt{x}) + \sqrt{2}x$

(d) $f(t) = \frac{3x^7 - \sqrt{x}}{x^2}$

(e) $g(x) = 8\left(\frac{e^x}{5} - \frac{5}{x^2+1}\right)$

(f) $s(t) = \frac{9t^3 - 4t^{5/3} - 4}{t}$

5. Given $f'(x) = x\sqrt{x}$ and $f(1) = 2$, find $f(x)$. **Note:** The directions are different here. You are not asked to find a *family* of antiderivatives but a *particular* antiderivative.

6. Explain *geometrically* what piece of information you are given in the previous problem that allows you to identify a particular member of the family of antiderivatives.

7. Find (the particular function) $f(x)$ assuming:

   - $f''(x) = \sqrt[3]{x}$
   - $f'(8) = 1$ and $f(1) = -6$.

8. A particle moves in a straight line and has acceleration given by $a(t) = 5\cos t - 2\sin t$. Its initial velocity is $v(0) = -6$ m/s and its initial position is $s(0) = 2$ m. Find its position function $s(t)$.

9. A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 feet above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground? (Hint: Acceleration due to gravity is $32\ ft\ /sec^2$.)